GCE AS/A level

0983/01

MATHEMATICS – S1
Statistics

A.M. MONDAY, 28 January 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The independent events A, B are such that

$$P(A) = 0{\cdot}2,\ P(A \cup B) = 0{\cdot}4.$$

(a) Determine the value of $P(B)$. [4]

(b) Calculate the probability that exactly one of the events A, B occurs. [3]

2. The random variable X has the binomial distribution B(16, 0·2). The random variable Y is defined by

$$Y = 2X + 5.$$

(a) Find the mean and variance of Y. [6]

(b) Evaluate $P(Y = 11)$. [3]

3. Bill buys a bag of sweets, of which 6 are red, 4 are green and 1 is blue. He selects 3 sweets at random, without replacement, from the bag. Calculate the probability that his selection contains

(a) 2 red sweets and another sweet of a different colour, [3]

(b) 2 sweets of the same colour and another sweet of a different colour. [5]

4. Customers arrive at a shop such that the number arriving during an interval of length t minutes has a Poisson distribution with mean $0{\cdot}1t$.

(a) Find the probability that, between 10 a.m. and 11 a.m.,

(i) exactly 4 customers arrive,

(ii) the number of arrivals lies between 2 and 8 (both inclusive). [6]

(b) Let X denote the number of customers arriving between 3 p.m. and 5 p.m.
Write down the value of $E(X)$ and hence find the value of $E(X^2)$. [3]

5. *(a)* When a certain type of seed is planted, there is a probability of 0·7 that it produces red flowers. A gardener plants 20 of these seeds. Calculate the probability that

(i) exactly 15 seeds produce red flowers,

(ii) more than 12 seeds produce red flowers. [6]

(b) When a different type of seed is planted, there is a probability of 0·09 that it produces white flowers. The gardener plants 150 of these seeds. Use an appropriate Poisson distribution to determine, approximately, the probability that exactly 10 seeds produce white flowers. [3]

6. The probability distribution of the discrete random variable X is given by

$$P(X = x) = k(1 + x) \qquad \text{for } x = 1, 2, 3, 4,$$
$$P(X = x) = 0 \qquad \text{otherwise.}$$

(a) Show that

$$k = \frac{1}{14}\,.$$ [2]

(b) Find the mean and variance of X. [5]

(c) Given that X_1, X_2 are independent observations on X, determine the value of $P(X_2 = 1 + X_1)$. [4]

7. In a mass screening programme, a new diagnostic test is being used to detect the presence or otherwise of a certain disease. When the person being tested has the disease, the test gives a positive result with probability 0·96. When the person being tested does not have the disease, the test gives a positive result with probability 0·01. It is known that 2% of the population have this disease. The test is given to a randomly chosen member of the population.

(a) Find the probability that a positive result is obtained. [3]

(b) Given that a positive result is obtained, find the probability that

(i) this person has the disease,

(ii) a positive result will be obtained if a second test is given to this person. [6]

8. The continuous random variable X has cumulative distribution function F given by

$$F(x) = 0 \qquad \text{for } x < 0,$$
$$F(x) = 2x^2 - x^4 \qquad \text{for } 0 \leqslant x \leqslant 1,$$
$$F(x) = 1 \qquad \text{for } x > 1.$$

(a) (i) Evaluate $P(0{\cdot}25 \leqslant X \leqslant 0{\cdot}75)$.

(ii) Show that the median m satisfies the equation

$$2m^4 - 4m^2 + 1 = 0.$$

(iii) Hence find the value of m, giving your answer correct to 3 significant figures. [7]

(b) (i) Find an expression for $f(x)$, valid for $0 \leqslant x \leqslant 1$, where f denotes the probability density function of X.

(ii) Evaluate $E\left(\sqrt{X}\right)$. [6]